

I. Are there uncertainties that are not risks? ooo. — II. Uncertainties that are not risks, ooo. — III. Why are some uncertainties not risks, ooo?

Author: Please see questions on pp. 26+30.

Abstract of: "Risk, Ambiguity and the Savage Axioms"
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Although L.J. Savage's postulates of rational behavior under uncertainty seem valid both normatively and descriptively in many situations, a class of situations is discussed in this paper in which many reasonable people neither wish nor tend to conform to the Savage postulates, even upon reflection. It follows that there is no way to infer meaningful probabilities for events, even approximately or qualitatively, from their choices in these situations; nor can they be described as acting "as if" they were maximizing the mathematical expectation of utility, in terms of any probabilities whatever.

Following Frank Ramsey, L.J. Savage and others have proposed postulates of rational behavior under uncertainty that would imply that all uncertainties could be represented as "risks": i.e., that subjective, ~~qualitative~~ probabilities

Following Frank Ramsey, L.J. Savage and others have proposed postulates of rational behavior under uncertainty ~~that imply that~~ which imply that individuals obeying these constraints in their choices among uncertain gambles act "as if" they assigned numerical probabilities to relevant contingencies, and maximized the mathematical expectation of a certain utility index over outcomes; in fact, it is possible to infer or measure both probabilities and utilities for such individuals by observing their choices in a suitable set of gambles. Savage regards these postulates as normative criteria, to which he expects reasonable persons to wish to conform, and to which ~~in at least their reflective~~ they presumably tend to conform, at least in their more reflective behavior. ~~However, in this paper a large class of situations is discussed~~ However, without denying that ~~the Savage axioms seem acceptable both normatively and descriptively for many situations,~~ ^{Although L.J. Savage's postulates of rational behavior under uncertainty} ~~another~~ ⁱⁿ class of situations is discussed in this paper in which many ~~otherwise~~ ^{valid} reasonable people neither wish nor tend to conform to the Savage postulates, even upon reflection. It follows that there is simply no way to infer meaningful probabilities for events from their ~~situational~~ choices in these situations; ~~nor can they be described as maximizing the mathematical expectation of utility on the basis of numerical probabilities derived~~ ^{nor can they be described as acting "as if" they assigned numerical probabilities to events and maximizing the mathematical expectation of utility.} ~~in any other fashion.~~

For example, imagine an urn known to contain ~~in~~ 30 Red balls and 60 ~~balls in an~~ Black and Yellow balls, with the proportion of Black to Yellow balls unknown. One ball is to be drawn at random from the urn, and a payoff is specified depending on the color drawn and the action (gamble) chosen from the following two pairs, I and II, III and IV:

	30		60	
	Red	Black	Yellow	
I	\$100	\$0	\$0	
II	\$0	\$100	\$0	
III	\$100	\$0	\$100	
IV	\$0	\$100	\$100	

The most common preferences among these actions are: I preferred to II, IV preferred to III. Such persons are violating Savage's Postulate 2, his basic "Sure-thing Principle"; in effect, they are acting "as if" they regarded Red as "more likely than" Black, but at the same time regarded ~~it~~ "not-Red" as more likely than "not-Black", ~~thus violating making~~ ^{it is} impossible to infer even a qualitative probability relationship between Red and Black from their choices.

None of the familiar criteria for predicting or prescribing decision-making under uncertainty ~~is~~ (the Savage axioms, minimax, the Hurwicz minimax-maximax criterion, minimax regret) corresponds to this pattern of choices; yet the behavior ~~seems~~ ^{and it can be described in terms of a simple, specified decision rule.} is deliberate and orderly. Such self-consistent behavior violating the Savage axioms seems to occur in situations that can be described as highly "ambiguous": in which available information is in some vital respects scanty or obviously unreliable or conflicting, and expressed "confidence" in estimates of likelihoods is low. Although individuals may ~~not~~ be far from "completely ignorant" in these situations--their estimates of relative likelihoods may be very asymmetric--the ~~irrationally~~ ^{confidence} may regard ~~the overall weight of the evidence~~ ^{as an inadequate basis of for decision} these estimates as unsure, inadequately supported. In reaching a decision, under these circumstances, many people seem to ~~give some weight to the question, "What is the worst distribution of payoffs that might reasonably be associated with this action?": the relative influence of this criterion depending on the confidence they place in their estimated, or "best guess" probability distribution.~~ ~~Indifferently~~ Such people act conservatively, (not pessimistically) without actually expecting the worst, in situations of high ambiguity; ^(the extent of this "bias" depending on their degree of confidence in their estimates) they choose to act "as if" the worse outcomes were somewhat more likely than their best estimates of likelihood would indicate. A simple decision rule can be specified which describes their behavior; one effect is to bias their choices ~~away from~~ toward familiar strategies, if available, with "known risks," as opposed to innovations whose outcomes are more ambiguous. Although the decision rule will lead to violations of the Savage axioms in ambiguous situations, their behavior is not clearly unreasonable. In any case, it is deliberate; for their behavior in these situations, the Bayesian or Savage approach gives wrong predictions and, by their lights, bad advice.

7 Having defined a "theory of reasoning" as a "theory of logic plus a theory of probability," Good defines a "reasonable body of beliefs" as "A body of beliefs which does not give rise to a contradiction when combined with a theory of reasoning."²

2. Good, "Rational Decisions," p. 109.

On the question of selecting a suitable theory of probability, he ~~makes~~ the important comment that, in addition to being consistent in terms of ^{the given} ~~your~~ theory of logic, "it must never force you into a ~~position~~ that after mature consideration you regard as untenable. (This would happen if a body of beliefs became classified as "unreasonable" [i.e., exhibited a "contradiction" in terms of the ~~given~~ theory of logic and the given theory of probability_] while not containing any judgments that could be conscientiously removed."³ In other words, if your tentative theory of probability

3. Good, Probability and the Weighing of Evidence, p. 5.

put in Chap. I

carries the implication that one of your beliefs "contradicts" another, and if ^{disavow or} after mature consideration you cannot conscientiously reject or modify either of these beliefs: then it is the theory you must discard, as unacceptable or invalid for you, for, as Savage puts it, "that kind of discrepancy seems intolerable in a normative theory."¹ The word "conscientious" is significant here. Elsewhere Good remarks

1. Savage, Foundations of Statistics, p. 102.

that "If a body of beliefs is found to be unreasonable after applying the abstract theory, then a good method of patching it up is by being honest (using unemotional judgment)"², and again, when "inconsistencies" are found in a body of beliefs in the

2. Good, "Rational Decisions," p. 111.

light of a given theory of probability, "you attempt to remove them by means of more mature judgment."³ When in the course of applying "honest, unemotional, mature

3. Good, "Subjective Probability as the Measure of a Non-Measurable Set,"

judgment" to imputed inconsistencies in your beliefs (or actions) you find ~~it easy~~ ~~yourself uncomfortable~~ it "not at all difficult to reverse one of them" without feeling dishonest but, on the contrary, feeling that you "have corrected an error"⁴ ~~(as Savage)~~ then the theory is not only confirmed anew but demonstrates its

4. Savage, Foundations of Statistics, pp. 21, 103; Savage asserts that these are his typical reactions when confronted with an apparent inconsistency in his own choices in terms of his theory of choice, and specifically in the case of an example proposed by Allais (p. 103).

usefulness. But by the same token, when the same sort of careful reflection does not suggest to you a way honestly to modify your expressed beliefs to make them "consistent" in terms of the theory: you must take a hard look at the theory. This is indeed a moment of truth. If the theory has served you well up to this moment,

~~can cannot be criticized~~ an ~~extreme~~ extreme reluctance to jettison it or even tamper with it is most understandable; yet one must be wary of the temptation to harden one's heart and pluck out the offending belief (or stated preference) to save the theory, when one cannot do so "conscientiously." A normative theory is needed, and loyalty to a familiar, proven one is natural; but blind unwillingness to submit it to this sort of test stands in the way of discovering ever a better one.

I have expanded on this subject, needless to say, because I believe that the Savage postulates to be considered later will fail, for many people, to pass this test; and I am trying to undermine in advance the instinct of some of them to or forcibly repress --all for a higher purpose-- falsify/or disavow non-conscientiously/their true intuitive perceptions of their beliefs or preferences, ~~for a higher purpose, i.e., to do so~~ "because otherwise they would be violating the theory." ~~At the same time,~~ ~~At the same time,~~